# The Crystal Structure of $\alpha$-HMX and a Refinement of the Structure of $\beta$-HMX* 

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The crystal structure of the $\alpha$ form of octahydro-1,3,5,7-tetranitro-1,3,5,7-tetrazocine, commonly known as cyclotetramethylene-tetranitramine or $\alpha-H M X$, has been determined by single-crystal X-ray methods. The unit cell is orthorhombic, space group Fdd2 with

$$
a=15 \cdot 14, b=23 \cdot 89, c=5 \cdot 913 \AA .
$$

There are eight molecules in the unit cell. The molecules have a basketlike shape with twofold symmetry. Bond lengths and angles all have normal values, but there are two rather short intermolecular $\mathrm{C}-\mathrm{O}$ distances of 3.04 and $3.20 \AA$.

## Introduction

The compound octahydro-1,3,5,7-tetranitro-1,3,5,7tetrazocine is a well known explosive commonly called cyclotetramethylene-tetranitramine or HMX. There are four known polymorphic forms, the optical properties of which have been described by McCrone (1950). The crystallographic data on these forms are summarized in Table 1. The stable form at room temperature is $\beta$-HMX. The infrared spectra of $\alpha, \gamma$ and $\delta$-HMX are similar to each other but are substantially different from that of $\beta$-HMX in the longer wavelength regions (Cady \& Smith, 1961). In part I of this paper we give the crystal structure of $\alpha$-HMX. In part II we present the results of a further refinement of the structure of $\beta$-HMX, based on the data of Eiland \& Pepinsky (1955), hereafter referred to as EP.

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## Part I.

Crystal structure of $\alpha-\mathrm{HMX}$

## Experimental

The temperature range in which $\alpha$-HMX is stable is approximately 103 to $162{ }^{\circ} \mathrm{C}$ (Cady \& Smith, 1961 ). A mixture of tris ( $\beta$-chloroethyl) phosphate and $\beta$-HMX was heated, with stirring, at $140{ }^{\circ} \mathrm{C}$ for about 24 hours. During this time complete conversion to $\alpha$-HMX occurred and a few single crystals of suitable size and quality for X-ray work were obtained. The length, parallel to the $c$ axis, of the selected crystal was about $200 \mu$. It had a triangular cross section bounded by ( 010 ), ( 110 ) and ( $1 \overline{1} 0$ ). The width of the base of this triangle, which was the (010) cleavage face, was about $100 \mu$ and its height was about $80 \mu$.

Precession and Weissenberg photographs were taken of this crystal and the space group was uniquely established as $F d d 2$ by systematic extinctions. Lattice

Table 1. Summary of crystallographic data on $H M X$ polymorphs

| Form | Space group | $a$ | $b$ | $c$ | $\beta$ | Z | $d_{m}$ | $d_{c}$ | Reference |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\alpha$ | $F d d 2$ | 15.14 | 23.89 | 5.913 | - | 8 | 1.84 | 1-839 | This work |
|  | $F d d 2$ | $15 \cdot 14$ | 23.75 | $5 \cdot 92$ | - | 8 | 1.83 | 1.848 | Claringbull \& Small (1961) |
|  |  | $15 \cdot 17$ | 24.03 | $5 \cdot 92$ | - | 8 |  | 1.823 | Armour Research Foundation (1958) |
| $\beta$ | $P 2_{1} / n$ | 6.54 | 11.05 | $7 \cdot 37$ | 102.8 | 2 | 1.90 | 1.894 | Eiland \& Pepinsky (1955) |
|  |  | $6 \cdot 50$ | $10 \cdot 93$ | 7.32 | 103.3 | 2 | 1.91 | 1.943 | Llewellyn \& Small (1961) |
| $\gamma$ | Pc or |  |  |  |  |  |  |  |  |
|  | $P 2 / \mathrm{c}$ or | 10.95 | 7.93 | 14.61 | 119.4 | 4 |  | 1.78 | Cady \& Smith (1961) |
|  | $P 2 / n$ | 13.28 | 7.93 | 10.95 | 106.5 | 4 |  | 1.78 |  |
|  | $P 2 / n$ | 13.15 | 7.93 | 10.93 | 105.5 | 4 | 1.76 | 1.79 | Claringbull \& Small (1961) |
| $\delta$ | $\begin{aligned} & P 6_{1} 22 \text { or } \\ & P 6_{5} 22 \end{aligned}$ | $7 \cdot 66$ | - | $32 \cdot 49$ | - | 6 | 1.80 | 1.786 | Claringbull \& Small (1961) |
|  |  | $7 \cdot 69$ | - | $32 \cdot 67$ | - | 6 |  | 1.764 | Armour Research Foundation |

constants were measured on a carefully aligned General Electric Co. single-crystal orienter and found to be

$$
\begin{aligned}
a= & 15 \cdot 14, b=23 \cdot 89, c=5 \cdot 913 \AA \\
& \left(\lambda \text { Mo } K \alpha_{1}=0.70926 \AA\right) .
\end{aligned}
$$

With $Z=8$, the calculated density is $1.839 \mathrm{~g} . \mathrm{cm}^{-3}$. Literature values for the density range between 1.83 (Claringbull \& Small, 1961) and $1.87 \mathrm{~g} . \mathrm{cm}^{-3}$ (McCrone, 1950).

Intensity measurements were made with Mo $K \alpha$ radiation and a single-crystal orienter equipped with a scintillation counter. The crystal was mounted on its $a$ axis. Background corrections were made by the balanced filter technique. Reflections within a sphere limited by $2 \theta \leq 40^{\circ}$ were examined. This sphere included reflections somewhat beyond the detection limit by film techniques. Since one half of the hemisphere was investigated, most of the equivalent reflections were measured twice. The agreement between equivalent values of $\left|F_{h k l}\right|^{2}$, expressed as

$$
\Sigma\left|\left|F_{H}\right|^{2}-\left|F_{H}\right|^{2}\right| / \Sigma\left(\left|F_{H}\right|^{2}+\left|F_{H^{\prime}}\right|^{2}\right),
$$

where $H=h k l$ and $H^{\prime}=h k l$, was $4 \cdot 8 \%$. Within this sphere, 261 non-equivalent reflections were observed out of a possible 282. The usual $L p$ corrections were applied to give a set of observed $\left|F_{n k l}\right|^{2}$ values on a relative scale. No absorption corrections were applied.

## Determination and refinement of the structure

There are eight molecules in the unit cell. Space group $F d d 2$ has a sixteenfold general position so that one half of the molecule is the asymmetric unit. The molecule is therefore required to have twofold symmetry, and the axis of the molecule must lie on a twofold symmetry axis. Because $F d d 2$ is a polar space group, the position of the molecule along the $z$ axis is chosen arbitrarily. Fisher-Hirschfelder molecular models were assembled. These models could be formed into a large variety of shapes without breaking the bonds. These shapes ranged from the centric molecule found in $\beta$-HMX to a basketlike molecule with symmetry $4 m$. Two different molecular models with twofold symmetry could be packed into the unit cell. One model, which subsequently proved to be correct, was basket shaped with all nitro groups on the same side of the ring. The other model had adjacent nitro groups on opposite sides of the ring.
A three-dimensional sharpened Patterson synthesis was computed. Only the first of the above models was compatible with the Patterson synthesis. Also, the orientation of the molecule with respect to rotation about the $z$ axis could be established. This orientation was within the range estimated by packing.

Approximate $x$ and $y$ coordinates of the atoms, other than the hydrogen atoms, were determined from the $z=0$ section of the sharpened Patterson. Estimates of the $z$ coordinates were determined from

| Table 2. Final $\alpha-H M X$ least-squares parameters |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Atom | $x$ | $y$ | $z$ | $\begin{array}{r} B_{11}{ }^{B_{1}} \times 10^{5} \end{array}$ | $\begin{gathered} B_{22} \\ \times 10^{5} \end{gathered}$ | $\begin{array}{r} B_{33} 3_{3} \\ \times 10^{5} \end{array}$ | $\begin{gathered} B_{12} \\ \times 10^{5} \end{gathered}$ | $\begin{gathered} B_{13}{ }^{1} \times 10^{5} \end{gathered}$ | $\begin{array}{r} B_{23}{ }^{2} \\ \times 10^{6} \end{array}$ |  | B |
| $\mathrm{C}_{1}$ | $0 \cdot 0995 \pm 7$ | $-0.0416 \pm 4$ | 0.0 | $263 \pm 62$ | $151 \pm 27$ | $1753 \pm 423$ | $118 \pm 67$ | $-242 \pm 258$ | $-151 \pm 165$ |  |  |
| $\mathrm{C}_{2}$ | $0 \cdot 0660 \pm 7$ | $0 \cdot 0618 \pm 5$ | $-0.0140 \pm 24$ | $341 \pm 63$ | $107 \pm 22$ | $1724 \pm 446$ | $-84 \pm 55$ | $88 \pm 275$ | $65 \pm 147$ |  |  |
| $\mathrm{N}_{1}$ | $0 \cdot 0181 \pm 5$ | $0 \cdot 0599 \pm 3$ | $0 \cdot 1035 \pm 18$ | $300 \pm 56$ | $83 \pm 14$ | $1257 \pm 373$ | $1 \pm 45$ | $-227 \pm 213$ | $27 \pm 112$ |  |  |
| $\mathrm{N}_{2}$ | $0 \cdot 1230 \pm 5$ | $0-0153 \pm 3$ | $0.0590 \pm 18$ | $237 \pm 37$ | $94 \pm 16$ | $2052 \pm 297$ | $-98 \pm 46$ | $68 \pm 192$ | $-91 \pm 132$ |  |  |
| $\mathrm{N}_{3}$ | $0.0210 \pm 6$ | $-0.0726 \pm 3$ | $0.3264 \pm 17$ | $408 \pm 65$ | $118 \pm 16$ | $1901 \pm 479$ | $-35 \pm 50$ | $246 \pm 288$ | $122 \pm 145$ |  |  |
| $\mathrm{N}_{4}$ | $0 \cdot 2060 \pm 5$ | $0 \cdot 0266 \pm 3$ | $0 \cdot 1374 \pm 21$ | $281 \pm 50$ | $171 \pm 20$ | $1734 \pm 339$ | $-25 \pm 53$ | $-180 \pm 184$ | $-243 \pm 141$ |  |  |
| $\mathrm{O}_{1}$ | $-0.0493 \pm 5$ | $-0.0877 \pm 3$ | $0 \cdot 4138 \pm 20$ | $385 \pm 41$ | $225 \pm 16$ | $1820 \pm 254$ | $-169 \pm 44$ | $-35 \pm 177$ | $310 \pm 117$ |  |  |
| $\mathrm{O}_{2}$ | $0 \cdot 0903 \pm 5$ | $-0.0670 \pm 3$ | $0 \cdot 4272 \pm 19$ | $442 \pm 45$ | $295 \pm 21$ | $1988 \pm 262$ | $-123 \pm 45$ | $-904 \pm 239$ | $19 \pm 122$ |  |  |
| $\mathrm{O}_{3}$ | $0 \cdot 2563 \pm 4$ | $-0.0133 \pm 3$ | $0 \cdot 1712 \pm 21$ | $232 \pm 32$ | $175 \pm 17$ | $2279 \pm 226$ | $58 \pm 38$ | $-445 \pm 153$ | $93 \pm 113$ |  |  |
| $\mathrm{O}_{4}$ | $0.2245 \pm 4$ | $0 \cdot 0757 \pm 3$ | $0 \cdot 1717 \pm 21$ | $272 \pm 30$ | $169 \pm 14$ | $3638 \pm 280$ | $-191 \pm 37$ | $-137 \pm 177$ | $404 \pm 122$ |  |  |
| $\mathrm{H}_{1}$ | $\underset{(0.087)}{0.088} \pm 5$ | $\underset{(-0.042)}{-0.043} \pm 3$ | $\frac{-0.178}{(-0.175)} \pm 17$ |  |  |  |  |  |  |  | $\pm 1.8 \AA^{2}$ |
| $\mathrm{H}_{2}$ | $\underset{(0.157)}{0.142} \pm 6$ | $\frac{-0.062}{(-0.063)} \pm 4$ | $\underset{(0.035)}{0.038} \pm 16$ |  |  |  |  |  |  |  | $\pm 2 \cdot 3$ |
| $\mathrm{H}_{3}$ | $\begin{gathered} 0.060 \\ (0.070) \end{gathered} \pm 8$ | $\underset{(0.055)}{0.059} \pm 5$ | $\underset{(-0.180)}{-0.154} \pm 25$ |  |  |  |  |  |  |  | $\pm 3 \cdot 8$ |
| $\mathrm{H}_{4}$ | $\underset{(0.108)}{0.102} \pm 5$ | $\begin{gathered} 0.099 \\ (0.095) \end{gathered}$ | $\underset{(0.037)}{0.027} \pm 14$ |  |  |  |  |  |  |  | $\pm 1.8$ |

Table 3. Observed and calculated magnitudes of the structure factors for $\alpha-H M X$
The column headings are $\mathrm{K}, 10\left|F_{o}\right|$ and $10\left|F_{c}\right|$. If $\left|F_{o}\right|$ is negative the minus sign means 'less than'

the Fisher-Hirschfelder model. The coordinates were used as starting parameters in a least-squares refinement which minimized $\Sigma(\Delta F)^{2}$. Form factors were used in functional form with parameters given by Forsyth \& Wells (1959). The full matrix was used and all observations were given unit weight. Isotropic temperature factors were initially applied. The $z$ of $\mathrm{C}_{1}$ was arbitrarily fixed at 0 . The problem quickly converged and showed that the trial structure was correct. After four cycles the value of $R$ was $8 \cdot 9 \%$ including unobserved reflections. All atoms were then allowed to become anisotropic and $R$ was reduced to $5.5 \%$ with unobserved reflections omitted. A dif-ference-Fourier synthesis was then calculated in an attempt to locate the hydrogen atoms. Peaks of about 0.25 e. $\AA^{-3}$ in height were observed in reasonable locations. There were a few peaks of about $0 \cdot 20$ e. $\AA^{-3}$ but these were not in acceptable positions. The standard deviation of the electron density is estimated to be $0 \cdot 1$ e. $\AA^{-3}$ by Cruickshank's (1949) formula. The hydrogen atoms were not included in the phase calculations for the difference-Fourier synthesis; therefore the observed peak heights are about half their true value.

When hydrogen atoms in these positions were included in the calculated structure factors, $R$ was reduced to $3.9 \%$. An attempt was made to leastsquare isotropic hydrogen atoms with all other atoms anisotropic, and $R$ was further reduced to $3.5 \%$. The positions of the hydrogen atoms converged, but the temperature factors of three of these atoms tended to become negative. This anomalous behavior on the part of the hydrogen atoms probably arose
from the fact that we now had 106 parameters and only 261 observations, even though the observations were reasonably accurate.
The final least-squares parameters are given in Table 2. The hydrogen positions from the differenceFourier synthesis are given in parentheses in Table 2. The final observed and calculated structure factors, for which $R=3 \cdot 5 \%$ with unobserved reflections excluded, are given in Table 3. The calculated values in Table 3 were made with the least-squares parameters of Table 2. As fractions of the standard deviations, the maximum shift was 0.023 and the mean shift 0.0067 in the last cycle. These values do not include the hydrogen atoms.

## Discussion of the structure of $\alpha$-HMX

A drawing of the molecule and a portion of the unit cell as viewed along the twofold $z$ axis is shown in Fig. I. A view along the $y$ axis is shown in Fig. 2 and one along the $x$ axis in Fig. 3. Hydrogen atoms are not shown in these figures. The interatomic distances and angles are given in Table 4. Distances and angles involving hydrogen atoms located by the difference-Fourier synthesis are given in parentheses. Many of these distances and angles are also shown in Figs. I and 2. The basketlike shape of the molecule can be seen clearly in these figures.

The bond distances all agree well with normal values and are essentially the same as found in $\beta$-HMX. The only unusual features in the structure of $\beta$-HMX are the two rather short intermolecular O-C distances of 3.04 and $3.14 \AA$. In $\alpha$-HMX there


Fig. 1. $\alpha$-HMX molecule and a portion of the unit cell as viewed along the $z$ axis.


Fig. 2. $\alpha$-HMX molecule and a portion of the unit cell as viewed along the $y$ axis.


Fig. 3. $\alpha$-HMX molecule as viewed along the $x$ axis.

Table 4. Interatomic distances and angles in $\alpha$-HMX

| Bond | Distance ( $\AA$ ) |
| :---: | :---: |
| $\mathrm{C}_{1}-\mathrm{H}_{1}$ | $\underset{(1 \cdot 05)}{1 \cdot 07} \pm 0 \cdot 10$ |
| $\mathrm{C}_{1}-\mathrm{H}_{2}$ | $\underset{(1.02)}{0.83} \pm 0.09$ |
| $\mathrm{C}_{1}-\mathrm{N}_{1}$ | $1.445 \pm 0.013$ |
| $\mathrm{C}_{1}-\mathrm{N}_{2}$ | $1.447 \pm 0.013$ |
| $\mathrm{C}_{2}-\mathrm{H}_{3}$ | $\underset{(1.00)}{0.84} \pm 0.15$ |
| $\mathrm{C}_{2}-\mathrm{H}_{4}$ | $\underset{(1.06)}{1.07} \pm 0.07$ |
| $\mathrm{C}_{2}-\mathrm{N}_{1}{ }^{\prime}$ | $1.450 \pm 0.014$ |
| $\mathrm{C}_{2}-\mathrm{N}_{2}$ | $1.471 \pm 0.014$ |
| $\mathrm{N}_{3}-\mathrm{N}_{1}$ | $1.354 \pm 0.010$ |
| $\mathrm{N}_{3}-\mathrm{O}_{1}$ | $1.238 \pm 0.013$ |
| $\mathrm{N}_{3}-\mathrm{O}_{2}$ | $1.215 \pm 0.013$ |
| $\mathrm{N}_{4}-\mathrm{N}_{2}$ | $1.367 \pm 0.012$ |
| $\mathrm{N}_{4}-\mathrm{O}_{3}$ | $1.235 \pm 0.010$ |
| $\mathrm{N}_{4}-\mathrm{O}_{4}$ | $1.225 \pm 0.010$ |

Short intermolecular

| distances |  |
| :--- | :--- |
| $\mathrm{N}_{4}-\mathrm{O}_{3}{ }^{\prime}$ | $\mathbf{2 . 8 3 7} \pm 0.017$ |
| $\mathrm{C}_{2}-\mathrm{O}_{4}$ | $\mathbf{3 . 0 4 5} \pm 0.013$ |
| $\mathrm{C}_{1}-\mathrm{O}_{3}{ }^{\prime}$ | $\mathbf{3 . 2 0 3} \pm 0.015$ |

are also two short non-bonded $\mathrm{O}-\mathrm{C}$ distances, namely $\mathrm{C}_{2}-\mathrm{O}_{4}, 3 \cdot 04 \AA$, and $\mathrm{C}_{1}-\mathrm{O}_{3}^{\prime}, 3 \cdot 20 \AA$.

The crystals show good cleavage on (010). The reason for this is made clear from an inspection of Fig. 1. Only intermolecular bonds need be broken when the crystal is cleaved on the glide plane normal to $y$.

The atoms in the $\mathrm{N}-\mathrm{NO}_{2}$ groups are essentially co-planar. The equations of the two pertinent planes are, for $\mathrm{N}_{1}, \mathrm{~N}_{3}, \mathrm{O}_{1}$ and $\mathrm{O}_{2},-0.2301 X+0.9488 Y+$ $0 \cdot 2163 Z+1 \cdot 2965=0$, and for $\mathrm{N}_{2}, \mathrm{~N}_{4}, \mathrm{O}_{3}$ and $\mathrm{O}_{4}$, $-0.3346 X-0.0770 Y+0.9393 Z+0.3255=0$. The distances of the atoms from these planes are given in Table 5. The entire nitramine groups of six atoms are more nearly co-planar in $\alpha$-HMX than in $\beta$-HMX.

Table 5. Distances of atoms from the $\mathrm{N}-\mathrm{NO}_{2}$ planes


Table 6. Magnitudes and direction angles, relative to the crystallographic axes, of the principal axes of the vibration ellipsoids

| Atom | Axis $i$ | $B_{i}\left(\AA^{2}\right)$ | $\alpha\left({ }^{\circ}\right)$ | $\beta\left(^{\circ}\right.$ ) | $\gamma\left({ }^{\circ}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{N}_{1}$ | 1 | $2.9 \pm 0.5$ | $20 \pm 18$ | $91 \pm 18$ | $110 \pm 18$ |
|  | 2 | $1 \cdot 9 \pm 0 \cdot 3$ | $85 \pm 25$ | $14 \pm 56$ | $77 \pm 53$ |
|  | 3 | $1 \cdot 6 \pm 0.5$ | $71 \pm 18$ | $104 \pm 56$ | $24 \pm 36$ |
| $\mathrm{N}_{2}$ | 1 | $2 \cdot 6 \pm 0 \cdot 3$ | $56 \pm 22$ | $117 \pm 21$ | $134 \pm 34$ |
|  | 2 | $1 \cdot 4 \pm 0 \cdot 3$ | $47 \pm 12$ | $44 \pm 12$ | $86 \pm 10$ |
|  | 3 | $3 \cdot 1 \pm 0 \cdot 3$ | $62 \pm 24$ | $121 \pm 20$ | $44 \pm 34$ |
| $\mathrm{N}_{3}$ | 1 | $3 \cdot 9 \pm 0 \cdot 6$ | $19 \pm 20$ | $97 \pm 20$ | $72 \pm 25$ |
|  | 2 | $3 \cdot 0 \pm 0 \cdot 4$ | $96 \pm 29$ | $40 \pm 28$ | $51 \pm 27$ |
|  | 3 | $2 \cdot 2 \pm 0 \cdot 4$ | $108 \pm 16$ | $129 \pm 28$ | $44 \pm 27$ |
| $\mathrm{N}_{4}$ | 1 | $2 \cdot 8 \pm 0 \cdot 4$ | $30 \pm 25$ | $99 \pm 16$ | $119 \pm 24$ |
|  | 2 | $4 \cdot 2 \pm 0 \cdot 4$ | $92 \pm 13$ | $21 \pm 11$ | $111 \pm 11$ |
|  | 3 | $1 \cdot 9 \pm 0 \cdot 4$ | $60 \pm 25$ | $71 \pm 10$ | $37 \pm 21$ |
| $\mathrm{C}_{1}$ | 1 | $1.9 \pm 0.5$ | $31 \pm 23$ | $111 \pm 32$ | $69 \pm 57$ |
|  | 2 | $4 \cdot 1 \pm 0.5$ | $61 \pm 12$ | $36 \pm 15$ | $109 \pm 15$ |
|  | 3 | $2 \cdot 3 \pm 0 \cdot 5$ | $100 \pm 54$ | $63 \pm 28$ | $29 \pm 46$ |
| $\mathrm{C}_{2}$ | 1 | $3 \cdot 5 \pm 0 \cdot 5$ | $30 \pm 19$ | $120 \pm 20$ | $87 \pm 27$ |
|  | 2 | $2 \cdot 0 \pm 0.5$ | $63 \pm 19$ | $41 \pm 33$ | $118 \pm 46$ |
|  | 3 | $2 \cdot 5 \pm 0 \cdot 5$ | $79 \pm 31$ | $65 \pm 43$ | $28 \pm 46$ |
| $\mathrm{O}_{1}$ | 1 | $3 \cdot 1 \pm 0 \cdot 3$ | $32 \pm 13$ | $72 \pm 10$ | $64 \pm 21$ |
|  | 2 | $6 \cdot 0 \pm 0 \cdot 3$ | $116 \pm 6$ | $30 \pm 5$ | $77 \pm 5$ |
|  | 3 | $2 \cdot 2 \pm 0 \cdot 3$ | $108 \pm 19$ | $113 \pm 8$ | $30 \pm 19$ |
| $\mathrm{O}_{2}$ | 1 | $4 \cdot 9 \pm 0 \cdot 3$ | $42 \pm 7$ | $68 \pm 8$ | $124 \pm 7$ |
|  | 2 | $7 \cdot 1 \pm 0 \cdot 4$ | $110 \pm 7$ | $22 \pm 8$ | $82 \pm 5$ |
|  | 3 | $1 \cdot 6 \pm 0 \cdot 3$ | $55 \pm 6$ | $85 \pm 4$ | $36 \pm 7$ |
| $\mathrm{O}_{3}$ | 1 | $1 \cdot 6 \pm 0 \cdot 3$ | $30 \pm 8$ | $102 \pm 6$ | $62 \pm 8$ |
|  | 2 | $4 \cdot 1 \pm 0 \cdot 4$ | $82 \pm 17$ | $12 \pm 12$ | $81 \pm 29$ |
|  | 3 | $3 \cdot 6 \pm 0 \cdot 3$ | $119 \pm 9$ | $94 \pm 32$ | $29 \pm 13$ |
| $\mathrm{O}_{4}$ | 1 | $1 \cdot 4 \pm 0 \cdot 2$ | $38 \pm 6$ | $56 \pm 5$ | $77 \pm 5$ |
|  | 2 | $4 \cdot 1 \pm 0 \cdot 3$ | 126士 6 | $54 \pm 9$ | $56 \pm 10$ |
|  | 3 | $5 \cdot 9 \pm 0 \cdot 3$ | $80 \pm 7$ | $125 \pm 8$ | $37 \pm 10$ |

The anisotropic thermal parameters were transformed to obtain the axes of the ellipsoids of thermal
motion and the directions of these axes with respect to the crystallographic axes. The results are given

Table 7. Least-squares parameters for $\beta-H M X$

| Atom | $x$ | $y$ | $z$ | $\begin{array}{r} B_{11} \\ \times 10^{4} \end{array}$ | $\begin{gathered} B_{22} \\ \times 10^{4} \end{gathered}$ | $\begin{gathered} B_{33} \\ \times 10^{4} \end{gathered}$ | $\begin{gathered} B_{12} \\ \times 10^{4} \end{gathered}$ | $\begin{array}{r} B_{13} \\ \times 10^{4} \end{array}$ | $\begin{gathered} B_{23} \\ \times 10^{4} \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{C}_{1}$ | $\begin{aligned} & 0.8024 \pm 21 \\ & (0.810) \end{aligned}$ | $\begin{aligned} & 0.0656 \pm 10 \\ & (0.065) \end{aligned}$ | $\underset{(0.785)}{0.7817 \pm 15}$ | $275 \pm 44$ | $80 \pm 10$ | $203 \pm 24$ | $-49 \pm 35$ | $258 \pm 53$ | $-22 \pm 26$ |
| $\mathrm{C}_{2}$ | $\begin{aligned} & 0 \cdot 2442 \pm 22 \\ & (0 \cdot 245) \end{aligned}$ | $\begin{gathered} 0 \cdot 1144 \pm 11 \\ (0 \cdot 114) \end{gathered}$ | $\underset{(0.053)}{0.0553 \pm 15}$ | $294 \pm 49$ | $93 \pm 11$ | $196 \pm 25$ | $11 \pm 40$ | $241 \pm 57$ | $42 \pm 27$ |
| $\mathrm{N}_{1}$ | $\begin{aligned} & 0 \cdot 4055 \pm 20 \\ & (0 \cdot 404) \end{aligned}$ | $\frac{-0.0006 \pm 10}{(-0.004)}$ | $\begin{aligned} & 0.7081 \pm 14 \\ & (0.708) \end{aligned}$ | $383 \pm 48$ | $85 \pm 10$ | $215 \pm 24$ | $2 \pm 35$ | $288 \pm 54$ | $-26 \pm 25$ |
| $\mathrm{N}_{2}$ | $\begin{gathered} 0.6593 \pm 18 \\ (0.658) \end{gathered}$ | $\frac{-0.0260 \pm}{(-0.027)}$ | $\begin{aligned} & 0.7070 \pm 13 \\ & (0 \cdot 708) \end{aligned}$ | $285 \pm 41$ | $79 \pm 9$ | $196 \pm 21$ | $-2 \pm 30$ | $233 \pm 47$ | $22 \pm 22$ |
| $\mathrm{N}_{3}$ | $\frac{-0.0185 \pm 17}{(-0.021)}$ | $\underset{(0 \cdot 122)}{0 \cdot 1211} \pm 8$ | $\stackrel{-0.0376 \pm 12}{(-0.039)}$ | $332 \pm 42$ | $66 \pm 8$ | $192 \pm 21$ | $15 \pm 28$ | $334 \pm 49$ | $-16 \pm 20$ |
| $\mathrm{N}_{4}$ | $\frac{-0 \cdot 1081 \pm 20}{(-0 \cdot 100)}$ | $\underset{(0.201)}{0.2021} \pm 9$ | $\begin{aligned} & 0.0304 \pm 14 \\ & (0.032) \end{aligned}$ | $362 \pm 47$ | $72 \pm 9$ | $219 \pm 23$ | $39 \pm 32$ | $309 \pm 54$ | $13 \pm 23$ |
| $\mathrm{O}_{1}$ | $\underset{(0.311)}{0.3044} \pm 18$ | $\underset{(0.073)}{0.0719} \pm 9$ | $\begin{aligned} & 0.5808 \pm 13 \\ & (0.581) \end{aligned}$ | $434 \pm 42$ | $114 \pm 10$ | $233 \pm 21$ | $47 \pm 35$ | $154 \pm 47$ | $25 \pm 25$ |
| $\mathrm{O}_{2}$ | $\begin{gathered} 0 \cdot 2964 \pm 17 \\ (0 \cdot 299) \end{gathered}$ | $\underset{(-0.060)}{-0.0606 \pm} 9$ | $\underset{(0.761)}{0.7597} \pm 13$ | $360 \pm 40$ | $117 \pm 10$ | $280 \pm 23$ | $-5 \pm 32$ | $411 \pm 50$ | $-6 \pm 24$ |
| $\mathrm{O}_{3}$ | $\begin{aligned} & 0.0475 \pm 19 \\ & (0.044) \end{aligned}$ | $\underset{(0.246)}{0.2464} \pm 8$ | $\underset{(0 \cdot 183)}{0 \cdot 1816 \pm 13}$ | $579 \pm 47$ | $89 \pm 9$ | $258 \pm 22$ | $-11 \pm 35$ | $332 \pm 54$ | $-97 \pm 24$ |
| $\mathrm{O}_{4}$ | $\begin{aligned} & 0.6692 \pm 18 \\ & (0.669) \end{aligned}$ | $\begin{gathered} 0 \cdot 2197 \pm 8 \\ (0 \cdot 220) \end{gathered}$ | $\underset{(-0.063)}{-0.0650 \pm 13}$ | $397 \pm 42$ | $93 \pm 9$ | $285 \pm 22$ | $105 \pm 30$ | $391 \pm 51$ | $28 \pm 23$ |



Fig. 4. Drawing of $\beta$-HMX molecule showing interatomic distances and angles. The distances given by EP are shown in parentheses.
in Table 6. The directions of the major axes of the vibration ellipsoids bear no consistent relationship to the normals to the planes of the nitramine groups. However, the orientation of most of the ellipsoids is not well defined because of the rather large standard deviation of the thermal parameters.

## Part II. <br> Refinement of the structure of $\beta$-HMX

The data of EP for $\beta$-HMX (709 non-zero $F(h k l)$ ) were subjected to a least-squares refinement after transformation of the data from space group $P 2_{1} / n$ to the standard orientation of $P 2_{1} / c$. All details of the least-squares analysis were the same as for $\alpha$-HMX. For the new orientation,

$$
a=6.54, b=11 \cdot 05, c=8.70 \AA ; \beta=124 \cdot 3^{\circ}
$$

EP report a final $R$ of $18 \cdot 7 \%$. After least-squares refinement with anisotropic temperature factors, but not including hydrogen, $R$ was reduced to $13.0 \%$. Table 7 gives the final least-squares parameters and, in parentheses, our starting parameters which are the transformed parameters given by EP. Only the parameters $x_{\mathrm{C}_{1}}, x_{\mathrm{N}_{4}}, x_{\mathrm{o}_{1}}$, and $y_{\mathrm{N}_{1}}$ have changed by more than three standard deviations. As a fraction of the standard deviation, the maximum shift was 0.0012 , and the mean shift 0.0002 in the last cycle.

The bond distances and angles for $\beta$-HMX are shown in Fig. 4, in which the distances found by EP are shown in parentheses. The least-squares standard deviations of these bonds range between 0.012 and $0.020 \AA$. The two short intermolecular C-O distances found by EP changed from $3 \cdot 01$ and $3 \cdot 12 \AA$ to $3 \cdot 04$ and $3 \cdot 14 \AA$.

EP state that the groups

are planar. We find that $\mathrm{C}_{2}^{\prime}, \mathrm{N}_{2}, \mathrm{~N}_{1}, \mathrm{O}_{1}$, and $\mathrm{O}_{2}$ are essentially planar but that $\mathrm{N}_{3}$ is $0.15 \AA$ and $\mathrm{C}_{1}$ is $0.44 \AA$ from this plane. Similarly, $\mathrm{C}_{1}, \mathrm{~N}_{3}, \mathrm{~N}_{4}, \mathrm{O}_{3}$ and $\mathrm{O}_{4}$ are planar but $\mathrm{C}_{2}$ is $0.23 \AA$ from this plane.

Although the final $R$ for $\alpha$-HMX is very much lower than that for $\beta$-HMX, the standard deviations of the bond lengths are essentially the same. This illustrates the importance of having a large quantity of data as well as data of high accuracy. Application of Cruickshank's (1960) formulae to the results for both $\alpha$ and $\beta$-HMX shows that the errors in atom positions should be approximately the same for both polymorphs.

An IBM-704 was used for calculating the $\alpha$-HMX Patterson. All other calculations were made with an IBM-7090 using programs written by the authors.

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# Die Struktur des Kaliumpolyphosphats $\left(\mathrm{KPO}_{3}\right)_{x}$ 

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(Eingegangen am 28 Juni 1962)
The structure has been solved by means of a generalized Patterson function. It consists of polyphosphate chains of the same type as the silicate chains in diopside, and contains $K^{+}$ions with 7 and 8 oxygen neighbours respectively.

## Allgemeines

Beim Entwässern von $\mathrm{KH}_{2} \mathrm{PO}_{4}$ bildet sich Kaliumpolyphosphat $\left(\mathrm{KPO}_{3}\right)_{x}$, eine Verbindung, die folgende interessante Eigenschaften besitzt:

Es existieren von ihr mehrere Modifikationen, die
sich reversibel ineinander umwandeln (Thilo \& Dostál, 1959). Unter Quellung können durch Austausch gegen Kalium organische Kationen, wie z.B. $n$-Hexyl-ammonium-Ionen $\mathrm{C}_{6} \mathrm{H}_{13}$. $\mathrm{NH}_{3}^{+}$und Anilinium-Ionen $\mathrm{C}_{6} \mathrm{H}_{5} . \mathrm{NH}_{3}^{+}$, eingelagert werden, zum Teil vielleicht sogar reversibel (Weiss \& Michel, 1958). Es gibt zwei


[^0]:    * Work performed under the auspices of the United States Atomic Energy Commission.

